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We explore practical methods of carrying out Lagrange Multiplier tests for variance components in two models in which the derivatives needed for the test are identically zero at the restricted estimates, the random effects probit model and the stochastic frontier model. The techniques are illustrated with two applications.

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ABSTRACT

We explore practical methods of carrying out Lagrange Multiplier tests for variance components in two models in which the derivatives needed for the test are identically zero at the restricted estimates, the random effects probit model and the stochastic frontier model. The techniques are illustrated with two applications.

Key Words: LM test, score test, random effects, stochastic frontier

JEL Classification: C1, C4

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1. Introduction

The Lagrange Multiplier (LM) test has provided a standard means of testing parametric restrictions for a variety of models. Its primary advantage among the trinity of tests (LM, Likelihood Ratio (LR) , Wald) generally used in likelihood based inference is that the LM statistic is computed using only the results of the null, restricted model, which is usually simpler than the alternative, unrestricted model. If, under the null hypothesis, the parameter being tested lies on the boundary of the parameter space, an additional advantage of the LM test is that it will still have standard distributional properties, whereas the LR and Wald tests will not. The random effects linear regression (Greene, 2012, p. 376) is a prominent example; Breusch and Pagan's (1980) LM test for random effects in a linear model is based on pooled OLS residuals, while estimation of the alternative model involves generalized least squares either based on a two step procedure or maximum likelihood.

The LM test can be interpreted as a Wald test of the distance from zero of the first derivative vector of the log likelihood function (the score vector) of the unrestricted model evaluated at the restricted maximum likelihood estimates. An example that will help to focus ideas is a probit model with exponential heteroscedasticity:

$$y_i^* = \boldsymbol{\beta}'\mathbf{x}_i + \varepsilon_i,\ \varepsilon_i \sim N[0, \{\exp(\boldsymbol{\alpha}'\mathbf{z}_i)\}^2],\ y_i = 1[y_i^* > 0].$$

The log likelihood function for the unrestricted model is

$$\log L = \Sigma_i \log \Phi\{(2y_i - 1)\boldsymbol{\beta}'\mathbf{x}_i / \exp(\boldsymbol{\alpha}'\mathbf{z}_i)\},$$

where $\Phi(t)$ is the cumulative density function (cdf) of the standard normal distribution. The maximization of logL over $(\boldsymbol{\beta},\boldsymbol{\alpha})$ is somewhat more complicated than the maximization over $(\boldsymbol{\beta},\mathbf{0})$, which is the standard probit model. But, an LM test for the absence of heteroscedasticity ($\boldsymbol{\alpha}=\mathbf{0}$) is based on estimates of the latter model and is extremely simple to carry out using standard tools. [See, e.g., Greene (2012, p. 713).]

Our interest here is testing for random effects in the random effects probit model using the LM test. This model is, after the linear regression model, by far the leading application of the more general class of random effects models. But, despite the obvious simplicity of the restricted model, the standard probit model, the LM test for this model does not appear in the existing literature, One reason for this is that the usual parameterization of the model has the inconvenient feature that the score vector is identically zero at the restricted estimates. There are noted in the received literature a handful of other cases in which the score vector needed to compute the LM statistic is identically zero at the restricted estimates, which would seem to preclude using the LM test. [See Chesher (1984), Lee and Chesher (1986) and Kiefer (1982).] In addition to the random effects probit model, we also examine in passing the stochastic frontier model for cross section data. Both cases are examples of a problem that emerges when the parametric restriction in the null hypothesis puts the value of a variance parameter on the boundary of the parameter space. In the random effects model, the restriction is that the standard deviation of the random effect equals zero. In the stochastic frontier model, the hypothesis of no inefficiency is tested via the hypothesis that the standard deviation of the random variable that is identified as the inefficiency component equals zero.

While Chesher (1984), Lee and Chesher (1986) and Kiefer (1982) discuss a general theory of how to deal with score vectors that are zero under the null hypothesis, and despite what would seem to be broad appeal in a generation of applications, we have not been able to locate applications in the subsequent 25+ years of literature. In this note, we will provide what we expect to be some useful analytical expressions for the LM test for random effects in the random effects probit modeland the stochastic frontier model. We illustrate their use with two empirical applications. [Computations were carried out using NLOGIT 5 (Econometric Software, Inc. 2012). Data and command streams may be downloaded from http://people.stern.nyu.edu/wgreene/lmtest.zip.]

2. Two Models in Which the Scores are Identically Zero

The random effects ordered probit model and the generic basic form of the stochastic frontier model are the subjects of many received applications.

2.1 The Random Effects Probit Model

The random effects probit model is

$$
\begin{aligned}
&y_{it}^* = \boldsymbol{\beta}'\mathbf{x}_{it} + u_i + \varepsilon_{it};\ i=1,...,n;\ t=1,...,\mathrm{T}_i,\\
&y_{it} = 1[y_{it}^* > 0],\\
&\varepsilon_{it} \sim N[0,1^2],\\
&u_i \sim N[0,\sigma_u^2],\\
&E[\varepsilon_{it}\varepsilon_{js}] = 0,\ i \neq j, t \neq s,\\
&E[u_i u_j] = 0,\ i \neq j,\\
&E[\varepsilon_{it} u_j] = 0\ \forall j,t,s,
\end{aligned}
$$

where $\boldsymbol{\beta}$ and $\mathbf{x}_{it}$ are both $k\times1$ vectors. The log likelihood for a sample of n observations, conditioned on the unobserved heterogeneity, $\mathbf{u} = (u_1,u_2,\ldots,u_n)'$, is

$$\log L(\boldsymbol{\beta}) \,|\, (\mathbf{u}) = \sum_{i=1}^{n} \log \prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + u_i)]$$

where $q_{it} = 2y_{it} - 1$. Maximum likelihood estimation is based on the unconditional log likelihood given by

$$\log L(\beta,\sigma_u) = \sum_{i=1}^{n} \log \int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + u_i)]\right\} \frac{1}{\sigma_u}\phi\left(\frac{u_i}{\sigma_u}\right) du_i,$$

where$\phi(t)$ is the standard normal density. The computation is simplified by making the change of variable from u_i to $v_i = u_i/\sigma_u$; the resulting log likelihood is

$$\log L(\boldsymbol{\beta},\sigma_u) = \sum_{i=1}^{n} \log L_i(\boldsymbol{\beta},\sigma_u) = \sum_{i=1}^{n} \log \int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + \sigma_u v_i)]\right\} \phi(v_i)\, dv_i.$$

Butler and Moffitt (1982) developed the estimation methods generally used in contemporary applications of this model.

To form the LM statistic for the test of the null hypothesis of no random effects, $\sigma_u = 0$, we require the derivative with respect to σ_u of each term in the sum:

$$\frac{\partial \log L_i(\beta,\sigma_u)}{\partial \sigma_u} = \frac{\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + \sigma_u v_i)]\right\} \left\{\sum_{t=1}^{T_i} \frac{\phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + \sigma_u v_i)]}{\Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + \sigma_u v_i)]}\right\} q_{it} v_i \phi(v_i)\, dv_i}{\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + \sigma_u v_i)]\right\} \phi(v_i)\, dv_i}$$

In order to compute the LM statistic, we need to evaluate this expression at $\sigma_u = 0$. Moving all terms not involving v_i outside the integrals produces very simple integrals in both the numerator and denominator.

$$\frac{\partial \log L_i(\boldsymbol{\beta},\sigma_u)}{\partial \sigma_u \left|(\sigma_u = 0)\right.} = \frac{\left\{\prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it})]\right\}\left\{\sum_{t=1}^{T_i} \frac{\phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it})]}{\Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it})]} q_{it}\right\}\int_{-\infty}^{\infty} v_i \phi(v_i)\, dv_i}{\left\{\prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it})]\right\}\int_{-\infty}^{\infty} \phi(v_i)\, dv_i}$$

Given the assumed standard normal distribution for v_i, the integral in the numerator is $E[v_i] = 0$ and that in the denominator is $\int_{v_i} \phi(v_i) dv_i = 1$ by definition. It follows that regardless of the value of $\boldsymbol{\beta}$ and the data,$\mathbf{X}$, each derivative term in the derivative of the log likelihood with respect to σ_u is identically zero. The derivatives with respect to the restricted MLE of $\boldsymbol{\beta}$ is also zero (again by definition). Hence, the score vector under the null hypothesis is identically zero. It also follows that the information matrix will be singular – the row and column corresponding to σ_u are identically zero.

In this situation, Chesher (1984), Lee and Chesher (1986) and Cox and Hinley(1974) suggest reparameterization of the model as a possible strategy for obtaining the LM test. For the probit model, we use $\gamma = \sigma_u^2$, so that the log likelihood in the parameter space of $(\boldsymbol{\beta},\gamma)$ becomes

$$\begin{aligned}\log L(\boldsymbol{\beta},\gamma) &= \sum_{i=1}^{n} \log L_i(\boldsymbol{\beta},\gamma) \\ &= \sum_{i=1}^{n} \log \int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]\right\}\phi(v_i)\, dv_i.\end{aligned}$$

The necessary derivative becomes

$$\begin{aligned}\frac{\partial \log L_i(\boldsymbol{\beta},\gamma)}{\partial \gamma} &= \frac{\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]\right\}\sum_{t=1}^{T_i}\left\{\frac{q_{it}\phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]}{\Phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]}\right\}\frac{1}{2\sqrt{\gamma}} v_i\phi(v_i)\, dv_i}{\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]\right\}\phi(v_i)\, dv_i} \\ &= \frac{\frac{1}{2\sqrt{\gamma}}\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]\right\}\left\{\sum_{t=1}^{T_i}\frac{q_{it}\phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]}{\Phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]}\right\} v_i\phi(v_i)\, dv_i}{\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]\right\}\phi(v_i)\, dv_i} \\ &= \frac{\frac{1}{2\sqrt{\gamma}}\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi_{it}\right\}\left\{\sum_{t=1}^{T_i} g_{it}\right\} v_i\phi(v_i)\, dv_i}{\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi_{it}\right\}\phi(v_i)\, dv_i}\end{aligned}$$

where $\Phi_{it} = \Phi\left[q_{it}\left(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma}\right)\right]$ and $g_{it} = \dfrac{q_{it}\phi\left[q_{it}\left(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma}\right)\right]}{\Phi\left[q_{it}\left(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma}\right)\right]}$.

Note that $g_{it}v_i$ is the first derivative of $\log\Phi_{it}$ with respect to $\sqrt{\gamma}$. Evaluated at $\gamma = 0$, using the same approach as earlier, the numerator now takes the form 0/0. We use L'Hôpital's rule to to evaluate the numerator, taking the limits as γ approaches zero from above. Then,

$$\frac{\partial \log L_i(\boldsymbol{\beta},\gamma)}{\partial\gamma|(\gamma=0)} = \frac{\lim_{\gamma\downarrow 0}\frac{1}{2\frac{1}{2\sqrt{\gamma}}}\int_{-\infty}^{\infty} L_i \left[\begin{matrix}\sum_{t=1}^{T_i}\left\{-\left(\frac{(q_{it}a_{it})\phi[q_{it}a_{it}]}{\Phi[q_{it}a_{it}]}\right)-\left(\frac{q_{it}\phi[q_{it}a_{it}]}{\Phi[q_{it}a_{it}]}\right)^2\right\} \\ +\left(\sum_{t=1}^{T_i} g_{it}\right)^2\end{matrix}\right]\frac{1}{2\sqrt{\gamma}}v_i^2\phi(v_i)\,dv_i}{\int_{-\infty}^{\infty} L_i\phi(v_i)\,dv_i}$$

$$= \frac{\lim_{\gamma\downarrow 0}\frac{1}{2\frac{1}{2\sqrt{\gamma}}}\int_{-\infty}^{\infty} L_i\left[\left(\sum_{t=1}^{T_i} h_{it}\right)+\left(\sum_{t=1}^{T_i} g_{it}\right)^2\right]\frac{1}{2\sqrt{\gamma}}v_i^2\phi(v_i)\,dv_i}{\int_{-\infty}^{\infty} L_i\phi(v_i)\,dv_i}$$

where $L_i = \prod_{t=1}^{T_i}\Phi_{it}$ and h_{it} is the second derivative of $\log\Phi_{it}$ with respect to its argument. The two occurrences of $1/(2\sqrt{\gamma})$ cancel. The integral in the numerator now involves $E[v_i^2] = 1$. Moving the now invariant (with respect to v_i) terms out of the integrals as before, the product terms, L_i, in the numerator and denominator cancel and we now have

$$\frac{\partial \log L_i}{\partial\gamma\,|\,(\gamma=0)} = \frac{1}{2}\sum_{t=1}^{T_i}\left\{-\left(\frac{q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}\phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}{\Phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}\right)-\left(\frac{q_{it}\phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}{\Phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}\right)^2\right\}+\frac{1}{2}\left[\sum_{t=1}^{T_i}\left(\frac{q_{it}\phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}{\Phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}\right)\right]^2$$

$$=\frac{1}{2}\left[\left(\sum_{t=1}^{T_i} h_{it}^0\right)+\left(\sum_{t=1}^{T_i} g_{it}^0\right)^2\right]$$

where the superscripts on h_{it} and g_{it} indicate they are evaluated at $\gamma = 0$. Under the null hypothesis, as T_i goes to infinity, each term (i) above would converge to zero by virtue of the information matrix inequality. To complete the derivation, the score for γ is

$$\frac{\partial \log L(\boldsymbol{\beta},\gamma)}{\partial\gamma|\ \ (\gamma=0)} = \frac{1}{2}\sum_{i=1}^{n}\left[\left(\sum_{t=1}^{T_i} h_{it}^0\right)+\left(\sum_{t=1}^{T_i} g_{it}^0\right)^2\right]$$

The remainder of the score vector at the restricted estimates is

$$\frac{\partial \log L(\boldsymbol{\beta},\gamma)}{\partial\boldsymbol{\beta}|(\gamma=0)} = \sum_{i=1}^{n}\left(\sum_{t=1}^{T_i} g_{it}^0\mathbf{x}_{it}\right)$$

Finally, collecting all K+1 terms, we denote the score vector as

$$\frac{\partial \log L(\boldsymbol{\beta},\gamma)}{\partial \begin{pmatrix} \boldsymbol{\beta} \\ \gamma \end{pmatrix} \Big| (\gamma = 0)} = \sum_{i=1}^{n} \mathbf{g}_i(\boldsymbol{\beta},0) = \sum_{i=1}^{n} \mathbf{g}_i^0$$

The first K elements of the score vector equal zero when evaluated at the restricted (pooled probit) MLE. Denote by $\mathbf{G}$ the $n\times(K+1)$ matrix with ith row equal to $\mathbf{g}_i^{0\prime}$ evaluated at the restricted maximum likelihood estimates, and let $\mathbf{i}$ denote an $n\times 1$ column vector of ones. Then, taking advantage of the information matrix equality to estimate the covariance matrix of the score vector, we compute the LM statistic using

$$LM = (\mathbf{i}'\mathbf{G})(\mathbf{G}'\mathbf{G})^{-1}(\mathbf{G}'\mathbf{i}) = \left(g_\gamma^0\right)^2 (\mathbf{G}'\mathbf{G})^{(K+1),(K+1)}$$

Where g_γ^0 is the last element of the score evaluated at the restricted maximum likelihood estimates, and $(\mathbf{G}'\mathbf{G})^{(K+1),(K+1)}$ is the $(K+1),(K+1)$ (i.e., lower right corner) element of $(\mathbf{G}'\mathbf{G})^{-1}$.

Given the well-known invariance of the LM test to re-parameterization (see Dagenais and Dufour (1981)), it might seem peculiar that a re-parameterization can change the properties of the test. However, their proof of invariance requires that the matrix containing the derivatives of one set of parameters with respect to the other set of parameters be non-singular at the restricted parameter values. Since $\partial\gamma/\partial\sigma_u=2\sigma_u$, this non-singularity condition will not be satisfied here at $\sigma_u=0$. Given the results for the parameterization using γ, it is easy to show that the parameterization using σ_u will lead to a zero score since

$$\frac{\partial \log L_i(\boldsymbol{\beta},\sigma_u)}{\partial\sigma_u} = \frac{\partial \log L_i(\boldsymbol{\beta},\gamma)}{\partial\gamma}\frac{\partial\gamma}{\partial\sigma_u}.$$

2.2 The Stochastic Frontier Model

The normal-half normal stochastic frontier model (Aigner, Lovell and Schmidt 1977) (ALS) is given by

$$y_i = \boldsymbol{\beta}'\mathbf{x}_i + v_i - u_i; i=1,..,n,$$

where $v_i \sim N[0,\sigma_v^2]$ and $u_i = |U_i|$ where $U_i \sim N[0,\sigma_u^2]$. It is assumed that the model contains a constant term. It is convenient to reparameterize the model at the outset, as do ALS, in terms of $\lambda = \sigma_u/\sigma_v$ and $\sigma^2 = \sigma_v^2 + \sigma_u^2$. Then, the log likelihood for the stochastic frontier model is

$$\log L(\boldsymbol{\beta},\sigma,\lambda) = \sum_{i=1}^{n} \log\left(\frac{2}{\sigma}\phi\left(\frac{\varepsilon_i}{\sigma}\right)\Phi\left(\frac{-\varepsilon_i\lambda}{\sigma}\right)\right)$$

where $\varepsilon_i = v_i - u_i$. The hypothesis of interest is that $\sigma_u = 0$, or, equivalently, $\lambda = 0$. The log likelihood for this model has two stationary points, one at the global MLE (with $\lambda > 0$) and a second at the point of interest, where $\lambda = 0$ and the estimator of $\boldsymbol{\beta}$ is simply the OLS coefficient vector. The score for λ is

$$\frac{\partial \log L(\boldsymbol{\beta},\sigma,\lambda)}{\partial\lambda} = \sum_{i=1}^{n} \frac{-\phi(\varepsilon_i\lambda/\sigma)}{\Phi(\varepsilon_i\lambda/\sigma)}\varepsilon_i.$$

It is easy to see that at $\lambda = 0$, the remaining MLEs of **β** and σ^2 are the ordinary least squares estimate of **β** and $s^2 = \mathbf{e}'\mathbf{e}/n$, where **e** is the vector of OLS residuals. The score for λ evaluated at the restricted parameter estimates yields $\phi(0)/\Phi(0) = (2/\pi)^{1/2} = .7979$ times the sum of the OLS residuals, which is zero. Reparameterization of the model, as we did for the probit model earlier, does not help here. However, Lee and Chesher (1984) obtained a result for testing $\lambda = 0$ based on higher moments. The statistic they derived is given by

$$LM = \sqrt{\frac{n}{6}}\frac{(1/n)\Sigma_{i=1}^{n}e_i^3}{\left(\sqrt{(1/n)\Sigma_{i=1}^{n}e_i^2}\right)^3} = \sqrt{\frac{n}{6}}\left(\frac{m_3}{s^3}\right) \xrightarrow{d} N[0,1],$$

where e_i is the OLS residual for the ith observation, m_3 is the third moment of the OLS residuals, and s is the standard deviation of the OLS residuals. The limiting distribution of LM^2 would be

$$LM^2 = \frac{n}{6}\left(\frac{m_3}{s^3}\right)^2 \xrightarrow{d} \chi^2[1].$$

The notable feature of this result is that this chi squared version of the test is just the skewness component of the Jarque-Bera (1987) test for normality based on the OLS residuals in a linear regression. (See also Bowman and Shenton's (1975) normality test.) It is of course natural to be testing for skewness in this context – the entire model hangs on the disturbance being distributed with a left skewed distribution. It is surprising that this statistic is not a standard part of the reported results for stochastic frontier models, since the likelihood ratio test of the same hypothesis usually is reported.

3. Applications

Riphahn, Wambach and Million (2003) used data from the German Socioeconomic Panel Survey over the period 1984-95 to model the number of times a patient visits a doctor in a year. Here, we restrict their sample to a balanced panel where the data are observed in each of the relevant years. This gives a panel data set with 7 years of data on 887 households for a total of 6209 observations.The variables used to modelthe decision of whether or not to visit a doctor in a calendar year (Y) are age (AGE), the years of schooling (EDUC), health satisfaction (HSAT), household income (HHINCOME), marital status (MARRIED), and a dummy variable for whether or not there are children under the age of 16 in the household (HHKIDS). The results of estimated the restricted pooled probit model and the random effects probit model are reported in Table 1.

Table 1: Estimated Probit Models

	Pooled Probit		Random Effect Probit	
	Coefficients	t-statistics	Coefficients	t-statistics
Constant	1.72326***	11.23	1.62003***	5.76
AGE	0.00399*	1.85	0.01334***	3.34
EDUC	-0.03943***	- 4.78	-0.04930***	-2.83
HSAT	-0.18107***	-21.61	-0.19446***	-16.98
HHINCOME	0.09388	0.86	0.1357	0.94
MARRIED	0.15919***	3.21	0.13719	1.62
HHKIDS	-0.13394***	-3.41	-0.11585*	-1.94
ρ			0.43203***	20.1
Log-Likelihood	-3760.5374		-3396.4481	

Note: *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

The computed value of the LM test is 244.2, which clearly rejects the null hypothesis of no random effects. The values of the Wald and LR tests are 403.8 and 728.2, respectively. Even if we take account of their non-standard distribution in this case (see Andrews (2001)), they also clearly reject the null hypothesis.

The second example is based on production data for a panel of 247 Spanish dairy farms used in Alvarez, Arias and Greene (2004). The data are the logs of milk output (YIT) and four inputs, labor (X1), land (X2), cows (X3) and feed (X4). The test statistic for skewness (21.67) clearly rejects the null hypothesis of zero inefficiency. Likewise, even taking account of their non-standard distributions, the Wald test (294.8) and the LR test (26.0) also clearly reject the null hypothesis.

Table 2: Estimated Stochastic Frontier Models

	Coefficients	t-statistics	Coefficients	t-statistics
Constant	11.5775***	3175.52	11.7014***	2614.87
X1	.59518***	30.39	.58369***	30.93
X2	.02305**	2.05	.03555***	3.2
X3	.02319*	1.78	.02256*	1.76
X4	.45176***	41.89	.44948***	43.42
λ	0		1.50164***	17.17
σ	.14012		.18710***	1698.9
Log-Likelihood	809.67609		822.68831	

Note: As for Table 1.

4. Conclusion

The test apparatus developed here appears in Lee and Chesher (1986) and Chesher (1984). (Though the LM test for the frontier model is presented in rather more opaque terms.) We find it surprising that neither appears to be in wide use, in spite of the fact that the null hypothesis being tested in both cases is routinely part of the analysis. The result for the stochastic frontier model has an intuitively appealing form. We have found no applications for the random effects probit model, despite its surprising simplicity. The result for the random effects case can actually be easily extended to other index function models. The function Φ(.) is the contribution to the likelihood function of the it'th observation and subsequent results are based on the first and second derivatives of logΦ(.).

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